

# Chapter 1 – Algebraic expressions

## ? Example 1 – Simplifying expressions

Simplify these expressions:

a.  $x^2 \times x^5$    b.  $2r^2 \times 3r^3$    c.  $\frac{b^7}{b^4}$    d.  $6x^5 \div 3x^3$    e.  $(a^3)^2 \times 2a^2$    f.  $(3x^2)^3 \div x^4$

## ? Example 3 – Simplifying fractions

Solve the following equations:

a.  $\frac{x^7 + x^4}{x^4}$    b.  $\frac{3x^2 - 6x^5}{2x}$    c.  $\frac{20x^7 - 15x^3}{5x^2}$

## ? Example 4 – Expanding double brackets

Expand these expressions and simplify if possible:

a.  $(x + 5)(x + 2)$    b.  $(x - 2y)(x^2 + 1)$    c.  $(x - y)^2$    d.  $(x + y)(3x - 2y - 4)$

## ? Example 5 – Expanding Trinomials

Expand these expressions and simplify if possible:

a.  $x(2x + 3)(x - 7)$    b.  $x(5x - 3y)(2x - y + 4)$    c.  $(x - 4)(x + 3)(x + 1)$

## ? Example 7 – Factorising quadratics

Factorise:

a.  $x^2 - 5x - 6$    b.  $x^2 + 6x + 8$    c.  $6x^2 - 11x - 10$    d.  $x^2 - 25$    e.  $4x^2 - 9y^2$

## ? Example 9 – Simplifying indices

Simplify:

a.  $\frac{x^3}{x^{-3}}$    b.  $x^{\frac{1}{2}} \times x^{\frac{3}{2}}$    c.  $(x^3)^{\frac{2}{3}}$    d.  $2x^{1.5} \div 4x^{-0.25}$    e.  $\sqrt[3]{125x^6}$    f.  $\frac{2x^2 - x}{x^5}$

## ? Example 10 – Fractional indices

Evaluate:

a.  $9^{\frac{1}{2}}$    b.  $64^{\frac{1}{3}}$    c.  $49^{\frac{3}{2}}$    d.  $25^{-\frac{3}{2}}$

## ? Example 11 – Indices – problem solving

Given that  $y = \frac{1}{16}x^2$  express each of the following in the form  $kx^n$ , where  $k$  and  $n$  are constants.

a.  $y^{\frac{1}{2}}$    b.  $4y^{-1}$

## ? Example 12 – Simplifying surds

Simplify:

a.  $\sqrt{12}$    b.  $\frac{\sqrt{20}}{2}$    c.  $5\sqrt{6} - 2\sqrt{24} + \sqrt{294}$

## ? Example 13 – Expanding brackets and surds

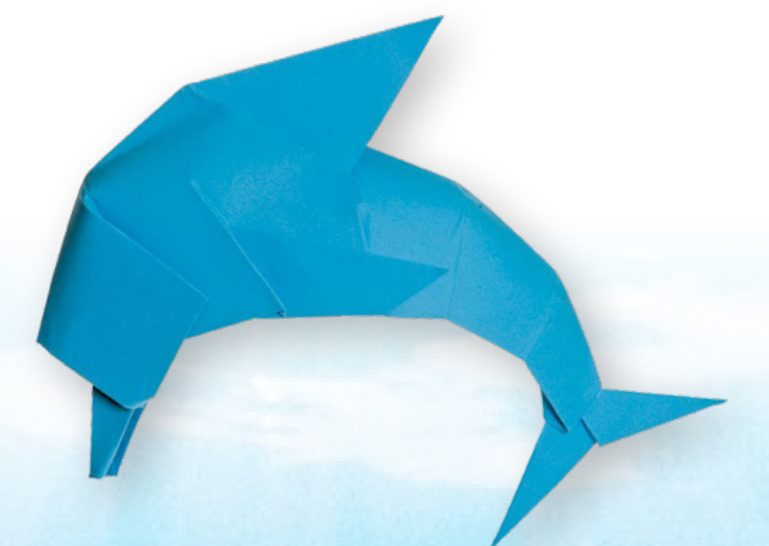
Expand and simplify if possible:

a.  $\sqrt{2}(5 - \sqrt{3})$    b.  $(2 - \sqrt{3})(5 + \sqrt{3})$

## ? Example 14 – Rationalising the denominator

Rationalise the denominator of:

a.  $\frac{1}{\sqrt{3}}$    b.  $\frac{1}{3 + \sqrt{2}}$    c.  $\frac{\sqrt{5} + \sqrt{2}}{\sqrt{5} - \sqrt{2}}$    d.  $\frac{1}{(1 - \sqrt{3})^2}$



# Chapter 2 – Quadratics

## ? Example 1 – Solving quadratic equations

Solve the following equations:

- a.  $x^2 - 2x - 15 = 0$       b.  $x^2 = 9x$       c.  $6x^2 + 13x - 5 = 0$   
d.  $x^2 - 5x + 18 = 2 + 3x$

## ? Example 2 – Solving quadratic equations without factorising

Solve the following equations:

- a.  $(2x - 3)^2 = 25$       b.  $(x - 3)^2 = 7$

## ? Example 3 – Quadratic formula to solve quadratic equations

Solve  $3x^2 - 7x - 1 = 0$  by using the formula.

## ? Example 4 – Completing the square

Complete the square for the expressions:

- a.  $x^2 + 8x$       b.  $x^2 - 3x$       c.  $2x^2 - 12x$

## ? Example 5 – Completing the square in the form $p(x + q)^2 + r$

Write  $3x^2 + 6x + 1$  in the form  $p(x + q)^2 + r$ , where  $p$ ,  $q$  and  $r$  are integers to be found.

## ? Example 6 – Solving quadratics by completing the square

Solve the equation  $x^2 + 8x + 10 = 0$  by completing the square.  
Give your answers in surd form.

## ? Example 7 – Solving quadratic equations

Solve the equation  $2x^2 - 8x + 7 = 0$ . Give your answers in surd form.

## ? Example 8 – Substituting into functions

The functions  $f$  and  $g$  are given by  $f(x) = 2x - 10$  and  $g(x) = x^2 - 9$ ,  $x \in \mathbb{R}$ .

- a. Find the values of  $f(5)$  and  $g(10)$ .  
b. Find the value of  $x$  for which  $f(x) = g(x)$ .

## ? Example 9 – Finding roots using functions

The function  $f$  is defined as  $f(x) = x^2 + 6x - 5$ ,  $x \in \mathbb{R}$ .

- a. Write  $f(x)$  in the form  $(x + p)^2 + q$ .  
b. Hence, or otherwise, find the roots of  $f(x)$ , leaving your answers in surd form.  
c. Write down the minimum value of  $f(x)$ , and state the value of  $x$  for which it occurs.

## ? Example 10 – Finding roots using functions

Find the roots of the function  $f(x) = x^6 + 7x^3 - 8$ ,  $x \in \mathbb{R}$ .

## ? Example 11 – Sketching quadratics

Sketch the graph of  $y = x^2 - 5x + 4$ , and find the coordinates of its turning point.

## ? Example 12 – Sketching quadratics without roots

Sketch the graph of  $y = 4x - 2x^2 - 3$ . Find the coordinates of its turning point and write down the equation of its line of symmetry.

## ? Example 13 – The discriminant: equal roots

Find the values of  $k$  for which  $f(x) = x^2 + kx + 9$  has equal roots.

## ? Example 14 – The discriminant: two distinct roots

Find the range of values of  $k$  for which  $x^2 + 4x + k = 0$  has two distinct real solutions.

## ? Example 15 – Modelling with quadratics

A spear is thrown over level ground from the top of a tower.

The height, in metres, of the spear above the ground after  $t$  seconds is modelled by the function:

$$h(t) = 12.25 + 14.7t - 4.9t^2, t \geq 0$$

- a. Interpret the meaning of the constant term 12.25 in the model.  
b. After how many seconds does the spear hit the ground?  
c. Write  $h(t)$  in the form  $A - B(t - C)^2$ , where  $A$ ,  $B$  and  $C$  are constants to be found.  
d. Using your answer to part c or otherwise, find the maximum height of the spear above the ground, and the time at which this maximum height is reached.

# Chapter 3 – Equations and inequalities

## ? Example 1 – Solving linear simultaneous equations by elimination

Solve the simultaneous equations:

a.  $2x + 3y = 8$   
 $3x - y = 23$

b.  $4x - 5y = 4$   
 $6x + 2y = 25$

## ? Example 2 – Solving linear simultaneous equations by substitution

Solve the simultaneous equations:

$x - y = 1$   
 $4x + 2y = -30$

## ? Example 3 – Solving simultaneous equations by substitution

Solve the simultaneous equations:

$2x + 2y = 3$   
 $x^2 + 3xy = 10$

## ? Example 4 – Linear simultaneous equations on graphs

a. On the same axes, draw the graphs of:

$2x + 3y = 8$   
 $3x - y = 23$

b. Use your graph to write down the solutions to the simultaneous equations.

## ? Example 5 – Non-linear simultaneous equations on graphs

a. On the same axes, draw the graphs of:

$2x + y = 3$   
 $y = x^2 - 3x + 1$

b. Use your graph to write down the solutions to the simultaneous equations.

## ? Example 6 – Non-linear simultaneous equations on graphs

The line with equation  $y = 2x + 1$  meets the curve with equation  $kx^2 + 2y + (k - 2) = 0$  at exactly one point. Given that  $k$  is a positive constant

- a. find the value of  $k$   
b. for this value of  $k$ , find the coordinates of the point of intersection.

## ? Example 7 – Linear inequalities

Find the set of values of  $x$  for which:

a.  $5x + 9 \geq x + 20$       b.  $12 - 3x < 27$       c.  $3(x - 5) > 5 - 2(x - 8)$

## ? Example 8 – Linear inequalities

Find the set of values of  $x$  for which:

a.  $3x - 5 < x + 8$  and  $5x > x - 8$       b.  $x - 5 > 1 - x$  or  $15 - 3x > 5 + 2x$

## ? Example 9 – Quadratic inequalities

Find the set of values of  $x$  for which:

$3 - 5x - 2x^2 < 0$

## ? Example 10 – Quadratic inequalities

- a. Find the set of values of  $x$  for which  $12 + 4x > x^2$   
b. Hence find the set of values for which  $12 + 4x > x^2$  and  $5x - 3 > 2$

## ? Example 11 – Non-linear inequalities

Find the set of values for which  $\frac{6}{x} > 2$ ,  $x \neq 0$

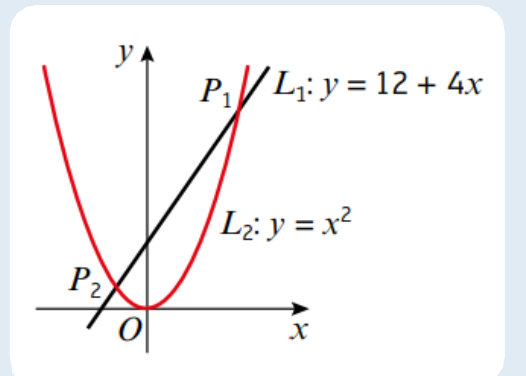
## ? Example 12 – Inequalities and simultaneous equations

$L_1$  has equation  $y = 12 + 4x$ .

$L_2$  has equation  $y = x^2$ .

The diagram shows a sketch of  $L_1$  and  $L_2$  on the same axes.

- a. Find the coordinates of  $P_1$  and  $P_2$ , the points of intersection.  
b. Hence write down the solution to the inequality  $12 + 4x > x^2$ .



## ? Example 13

On graph paper, shade the region that satisfies the inequalities:

$y \geq -2$ ,  $x < 5$ ,  $y \leq 3x + 2$  and  $x > 0$ .

## ? Example 14 – Non-linear equalities and regions of graphs

On graph paper, shade the region that satisfies the inequalities:

$2y + x < 14$   
 $y \geq x^2 - 3x - 4$

# Chapter 4 – Graphs and transformations

## ? Example 1 – Transformation of cubic functions

Sketch the curves with the following equations and show the points where they cross the coordinate axes.

**a.**  $y = (x - 2)(1 - x)(1 + x)$       **b.**  $y = x(x + 1)(x + 2)$

## ? Example 2 – Transformation of cubic functions

Sketch the following curves.

**a.**  $y = (x - 1)^2(x + 1)$       **b.**  $y = x^3 - 2x^2 - 3x$       **c.**  $y = (x - 2)^3$

## ? Example 3 – Transformation of cubic functions

Sketch the curve with equation  $y = (x - 1)(x^2 + x + 2)$ .

## ? Example 4 – Sketching quartics

Sketch the following curves:

**a.**  $y = (x + 1)(x + 2)(x - 1)(x - 2)$       **b.**  $y = x(x + 2)^2(3 - x)$   
**c.**  $y = (x - 1)^2(x - 3)^2$

## ? Example 5 – Sketching reciprocal graphs

Sketch on the same diagram:

**a.**  $y = \frac{4}{x}$  and  $y = \frac{12}{x}$       **b.**  $y = -\frac{1}{x}$  and  $y = -\frac{3}{x}$       **c.**  $y = \frac{4}{x^2}$  and  $y = \frac{10}{x^2}$

## ? Example 6 – Points of intersection of graphs

- a.** On the same diagram sketch the curves with equations  $y = x(x - 3)$  and  $y = x^2(1 - x)$ .  
**b.** Find the coordinates of the points of intersection.

## ? Example 7 – Points of intersection of graphs

- a.** On the same diagram sketch the curves with equations  $y = x^2(3x - a)$  and  $y = \frac{b}{x}$ , where  $a$  and  $b$  are positive constants.  
**b.** State, giving a reason, the number of real solutions to the equation  $x^2(3x - a) - \frac{b}{x} = 0$

## ? Example 8 – Points of intersection of graphs

- a.** Sketch the curves  $y = \frac{4}{x^2}$  and  $y = x^2(x - 3)$  on the same axes.  
**b.** Using your sketch, state, with a reason, the number of real solutions to the equation  $x^4(x - 3) - 4 = 0$ .

## ? Example 9 – Transformation of cubic functions

Sketch the graphs of:

**a.**  $y = x^2$       **b.**  $y = (x - 2)^2$       **c.**  $y = x^2 + 2$

## ? Example 10 – Translating graphs

$f(x) = x^3$

$g(x) = x(x - 2)$

Sketch the following graphs, indicating any points where the curves cross the axes:

- a.**  $y = f(x + 1)$   
**b.**  $y = g(x + 1)$

## ? Example 11 – Translating reciprocals

Given that  $h(x) = \frac{1}{x}$ , sketch the curve with equation  $y = h(x) + 1$  and state the equations of any asymptotes and intersections with the axes.

## ? Example 12 – Stretching quadratics

Given that  $f(x) = 9 - x^2$ , sketch the curves with equations:

**a.**  $y = f(2x)$       **b.**  $y = 2f(x)$

## ? Example 13 – Stretching cubics

- a.** Sketch the curve with equation  $y = x(x - 2)(x + 1)$ .  
**b.** On the same axes, sketch the curves  $y = 2x(2x - 2)(2x + 1)$  and  $y = -x(x - 2)(x + 1)$ .

## ? Example 14 – Reflecting quadratics

On the same axes sketch the graphs of  $y = f(x)$ ,  $y = f(-x)$  and  $y = -f(x)$  where  $f(x) = x(x + 2)$ .

# Chapter 5 – Straight line graphs

## ? Example 1 – Work out gradient given two points

Work out the gradient of the line joining  $(-2, 7)$  and  $(4, 5)$

## ? Example 2 – Calculations with gradients

The line joining  $(2, -5)$  to  $(4, a)$  has gradient  $-1$ . Work out the value of  $a$ .

## ? Example 3 – Alternative forms of straight-line equations

Write down the gradient and y-intercept of these lines:

a.  $y = -3x + 2$       b.  $4x - 3y + 5 = 0$

## ? Example 4 – Alternative forms of straight-line equations

Write these lines in the form  $ax + by + c = 0$

a.  $y = 4x + 3$       b.  $y = -\frac{1}{2}x + 5$

## ? Example 5 – Intersections with axes

The line  $y = 4x - 8$  meets the  $x$ -axis at the point  $P$ .

Work out the coordinates of  $P$ .

## ? Example 6 – Finding the equation of a straight line

Find the equation of the line with gradient 5 that passes through the point  $(3, 2)$ .

## ? Example 7 – Finding the equation of a straight line

Find the equation of the line that passes through the points  $(5, 7)$  and  $(3, -1)$ .

## ? Example 8 – Finding the equation of a straight line

The line  $y = 3x - 9$  meets the  $x$ -axis at the point  $A$ . Find the equation of the line with gradient  $\frac{2}{3}$  that passes through point  $A$ . Write your answer in the form  $ax + by + c = 0$ , where  $a$ ,  $b$  and  $c$  are integers.

## ? Example 9 – Finding the equation of a straight line

The lines  $y = 4x - 7$  and  $2x + 3y - 21 = 0$  intersect at the point  $A$ . The point  $B$  has coordinates  $(-2, 8)$ . Find the equation of the line that passes through the points  $A$  and  $B$ . Write your answer in the form  $ax + by + c = 0$ , where  $a$ ,  $b$  and  $c$  are integers.

## ? Example 10 – Equations of perpendicular lines

A line is parallel to the line  $6x + 3y - 2 = 0$  and it passes through the point  $(0, 3)$ . Work out the equation of the line.

## ? Example 11 – Testing for perpendicularity or parallelism

Work out whether these pairs of lines are parallel, perpendicular or neither:

a.  $3x - y - 2 = 0$       b.  $y = \frac{1}{2}x$   
 $x + 3y - 6 = 0$        $2x - y + 4 = 0$

## ? Example 12 – Finding the equations of perpendicular lines

A line is perpendicular to the line  $2y - x - 8 = 0$  and passes through the point  $(5, -7)$ .

Find the equation of the line.

## ? Example 13 – Working out the distance between two points

Find the distance between  $(2, 3)$  and  $(5, 7)$ .

## ? Example 14 – Working out distances and areas

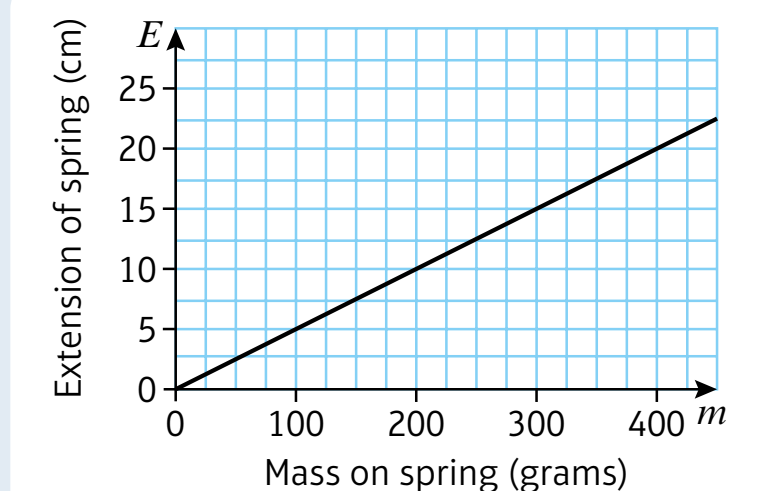
The straight line  $l_1$  with equation  $4x - y = 0$  and the straight line  $l_2$  with equation  $2x + 3y - 21 = 0$  intersect at point  $A$ .

- Work out the coordinates of  $A$ .
- Work out the area of triangle  $AOB$  where  $B$  is the point where  $l_2$  meets the  $x$ -axis.

## ? Example 15 – Modelling using straight line graphs

The graph shows the extension,  $E$ , of a spring when different masses,  $m$ , are attached to the end of the spring.

- Calculate the gradient,  $k$ , of the line.
- Write an equation linking  $E$  and  $m$ .
- Explain what the value of  $k$  represents in this situation.



# Chapter 6 – Circles

## ? Example 1 – Finding the centre of a circle

The line segment  $AB$  is a diameter of a circle, where  $A$  and  $B$  are  $(-3, 8)$  and  $(5, 4)$  respectively.

Find the coordinates of the centre of the circle.

## ? Example 2 – Problem solving with midpoints

The line segment  $PQ$  is a diameter of the circle centre  $(2, -2)$ . Given that  $P$  is  $(8, -5)$ , find the coordinates of  $Q$ .

## ? Example 3 – Problem solving with lines and circles

The line segment  $AB$  is a diameter of the circle centre  $C$ , where  $A$  and  $B$  are  $(-1, 4)$  and  $(5, 2)$  respectively. The line  $l$  passes through  $C$  and is perpendicular to  $AB$ . Find the equation of  $l$ .

## ? Example 4 – Finding the equation of a circle

Write down the equation of the circle with centre  $(5, 7)$  and radius 4.

## ? Example 5 – The equation of a circle

A circle has equation  $(x - 3)^2 + (y + 4)^2 = 20$ .

- Write down the centre and radius of the circle.
- Show that the circle passes through  $(5, -8)$ .

## ? Example 6 – Finding the equation of a circle

The line segment  $AB$  is a diameter of a circle, where  $A$  and  $B$  are  $(4, 7)$  and  $(-8, 3)$  respectively. Find the equation of the circle.

## ? Example 7 – Finding the equation of a circle

Find the centre and the radius of the circle with the equation  $x^2 + y^2 - 14x + 16y - 12 = 0$ .

## ? Example 8 – Intersection of linear graphs and circles

Find the coordinates of the points where the line  $y = x + 5$  meets the circle  $x^2 + (y - 2)^2 = 29$ .

## ? Example 9 – Problem solving with lines and circles

Show that the line  $y = x - 7$  does not meet the circle  $(x + 2)^2 + y^2 = 33$ .

## ? Example 10 – Finding the equation of a tangent to a circle

The circle  $C$  has equation  $(x - 2)^2 + (y - 6)^2 = 100$ .

- Verify that the point  $P(10, 0)$  lies on  $C$ .
- Find an equation of the tangent to  $C$  at the point  $(10, 0)$ , giving your answer in the form  $ax + by + c = 0$ .

## ? Example 11 – Problem solving using tangents and circles

A circle  $C$  has equation  $(x - 5)^2 + (y + 3)^2 = 10$ .

The line  $l$  is a tangent to the circle and has gradient  $-3$ .

Find two possible equations for  $l$ , giving your answers in the form  $y = mx + c$ .

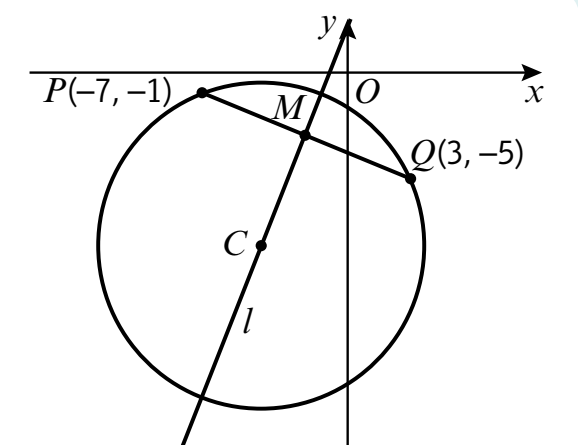
## ? Example 12 – Problem solving using chords and circles

The points  $P$  and  $Q$  lie on a circle with centre  $C$ , as shown in the diagram. The point  $P$  has coordinates  $(-7, -1)$  and the point  $Q$  has coordinates  $(3, -5)$ .

$M$  is the midpoint of the line segment  $PQ$ .

The line  $l$  passes through the points  $M$  and  $C$ .

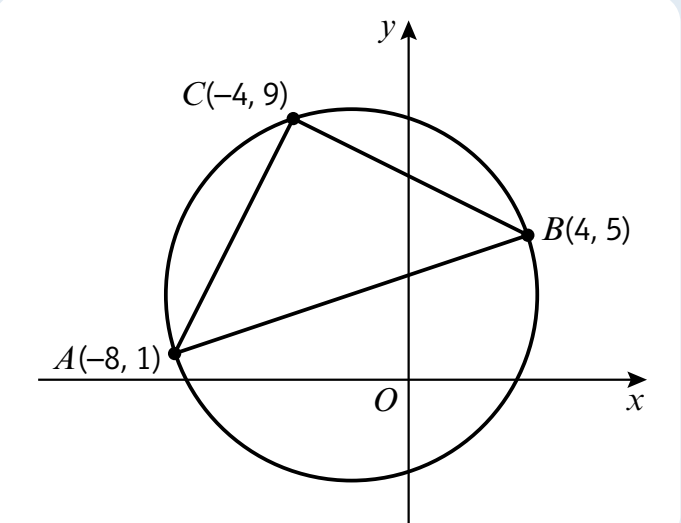
- Find an equation for  $l$ .
- show that the  $x$ -coordinate of  $C$  is  $-4$
- find an equation of the circle.



## ? Example 13 – Problem solving using circle theorems

The points  $A(-8, 1)$ ,  $B(4, 5)$  and  $C(-4, 9)$  lie on the circle, as shown in the diagram.

- Show that  $AB$  is a diameter of the circle.
- Find an equation of the circle.



## ? Example 14 – Finding the equation of a circle using chords

The points  $P(3, 16)$ ,  $Q(11, 12)$  and  $R(-7, 6)$  lie on the circumference of a circle. The equation of the perpendicular bisector of  $PQ$  is  $y = 2x$ .

- Find the equation of the perpendicular bisector of  $PR$ .
- Find the centre of the circle.
- Work out the equation of the circle.

# Chapter 7 – Algebraic methods

 **Example 1** – Simplifying algebraic fractions

Simplify these fractions:

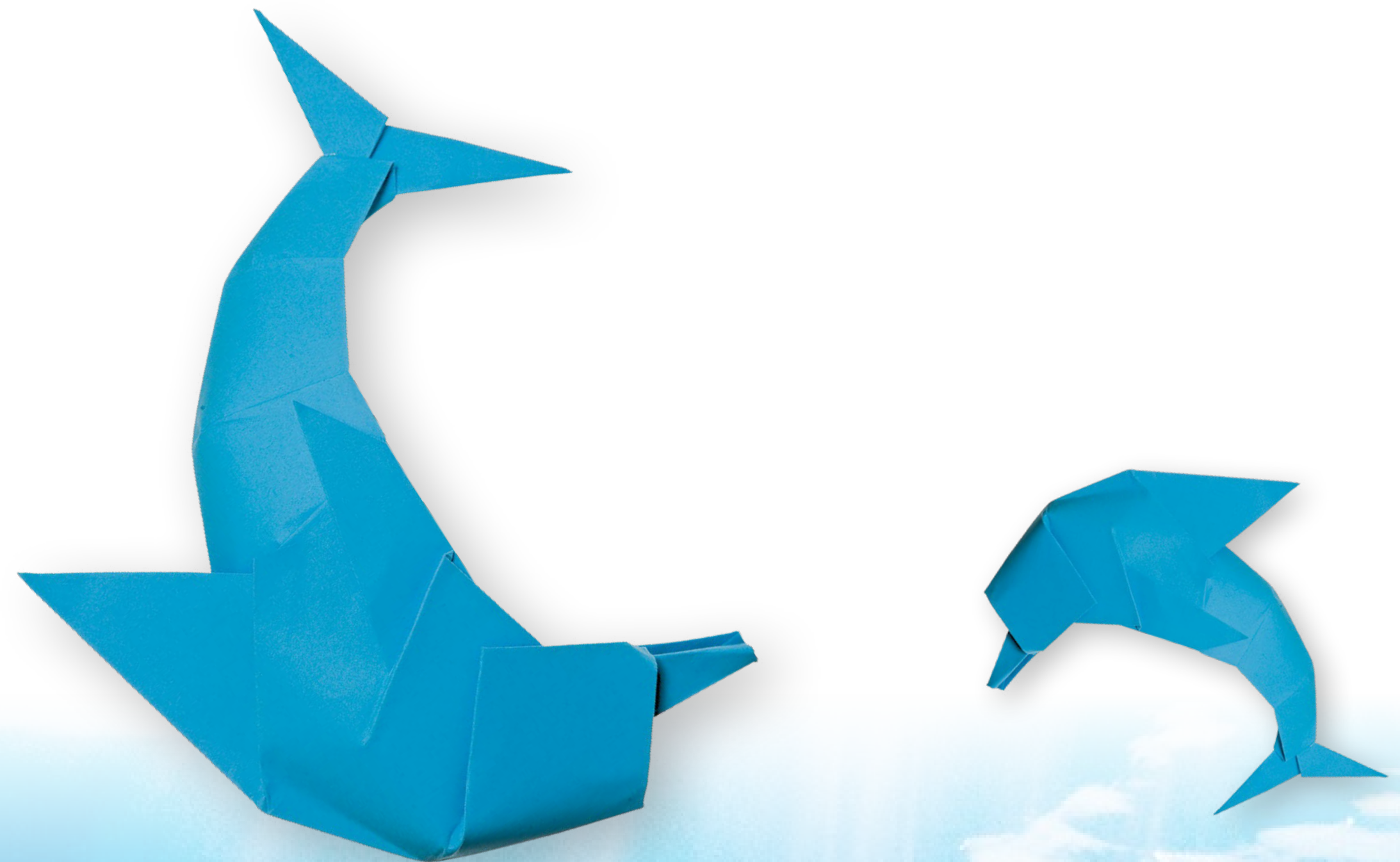
a.  $\frac{7x^4 - 2x^3 + 6x}{x}$

b.  $\frac{(x+7)(2x-1)}{(2x-1)}$

c.  $\frac{x^2 + 7x + 12}{(x+3)}$

d.  $\frac{x^2 + 6x + 5}{x^2 + 3x - 10}$

e.  $\frac{2x^2 + 11x + 12}{(x+3)(x+4)}$



# Chapter 9 – Trigonometric ratios

## ? Example 1 – The cosine rule to find missing sides

Calculate the length of the side  $AB$  of the triangle  $ABC$  in which  $AC = 6.5$  cm,  $BC = 8.7$  cm and  $\angle ACB = 100^\circ$ .

## ? Example 2 – The cosine rule to find missing angles

Find the size of the smallest angle in a triangle whose sides have lengths 3 cm, 5 cm and 6 cm.

## ? Example 3 – Bearings and the cosine rule

Coastguard station  $B$  is 8 km, on a bearing of  $060^\circ$ , from coastguard station  $A$ . A ship  $C$  is 4.8 km, on a bearing of  $018^\circ$ , away from  $A$ . Calculate how far  $C$  is from  $B$ .

## ? Example 4 – Problem solving using the cosine rule

In  $\triangle ABC$ ,  $AB = x$  cm,  $BC = (x + 2)$  cm,  $AC = 5$  cm and  $\angle ABC = 60^\circ$ . Find the value of  $x$ .

## ? Example 5 – The sine rule to find missing sides

In  $\triangle ABC$ ,  $AB = 8$  cm,  $\angle BAC = 30^\circ$  and  $\angle BCA = 40^\circ$ . Find  $BC$ .

## ? Example 6 – The sine rule to find missing angles

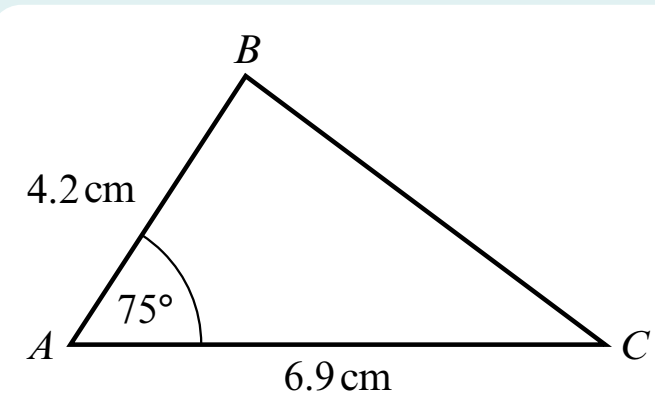
In  $\triangle ABC$ ,  $AB = 3.8$  cm,  $BC = 5.2$  cm and  $\angle BAC = 35^\circ$ . Find  $\angle ABC$ .

## ? Example 7 – The sine rule: the ambiguous case

In  $\triangle ABC$ ,  $AB = 4$  cm,  $AC = 3$  cm and  $\angle ABC = 44^\circ$ . Work out the two possible values of  $\angle ACB$ .

## ? Example 8 – The area of a triangle

Work out the area of the triangle shown below.



## ? Example 9 – Problem solving using the area of a triangle

In  $\triangle ABC$ ,  $AB = 5$  cm,  $BC = 6$  cm and  $\angle ABC = x$ . Given that the area of  $\triangle ABC$  is  $12$  cm<sup>2</sup> and that  $AC$  is the longest side, find the value of  $x$ .

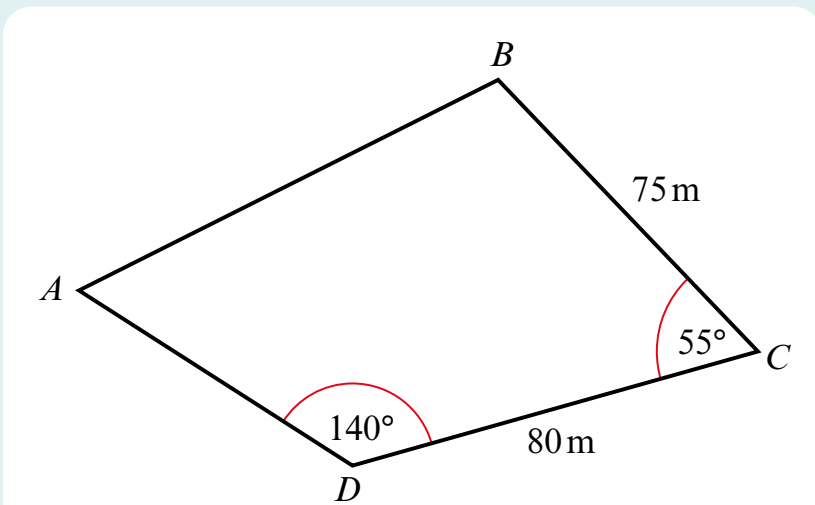
## ? Example 10 – Problem solving using the sine and cosine rules

The diagram shows the locations of four mobile phone masts in a field.  $BC = 75$  m,  $CD = 80$  m, angle  $BCD = 55^\circ$  and angle  $ADC = 140^\circ$ .

In order that the masts do not interfere with each other, they must be at least 70 m apart.

Given that  $A$  is the minimum distance from  $D$ , find:

- the distance  $A$  is from  $B$
- the angle  $BAD$
- the area enclosed by the four masts



## ? Example 11 – Sketching Trigonometric graphs

- Sketch the graph of  $y = \cos \theta$  in the interval  $-360^\circ \leq \theta \leq 360^\circ$ .
- Sketch the graph of  $y = \sin x$  in the interval  $-180^\circ \leq x \leq 270^\circ$
  - $\sin(-30^\circ) = -0.5$ . Use your graph to determine two further values of  $x$  for which  $\sin x = -0.5$ .

## ? Example 12 – Transforming trigonometric graphs

Sketch on separate sets of axes the graphs of:

- $y = 3 \sin x$ ,  $0 \leq x \leq 360^\circ$
- $y = -\tan \theta$ ,  $-180^\circ \leq \theta \leq 180^\circ$

# Chapter 10 – Trigonometric identities and equations

## ? Example 1 – Angles in all four quadrants

Write down the values of:

- a.**  $\sin 90^\circ$       **b.**  $\sin 180^\circ$       **c.**  $\sin 270^\circ$   
**d.**  $\cos 180^\circ$       **e.**  $\cos (-90^\circ)$       **f.**  $\cos 450^\circ$

## ? Example 2 – Angles in all four quadrants

Write down the values of:

- a.**  $\tan 45^\circ$       **b.**  $\tan 135^\circ$       **c.**  $\tan 225^\circ$   
**d.**  $\tan (-45^\circ)$       **e.**  $\tan 180^\circ$       **f.**  $\tan 90^\circ$

## ? Example 3 – Angles in all four quadrants

Find the signs of  $\sin \theta$ ,  $\cos \theta$  and  $\tan \theta$  in the second quadrant.

## ? Example 4 – Angles in all four quadrants

Express in terms of trigonometric ratios of acute angles:

- a.**  $\sin (-100^\circ)$       **b.**  $\cos 330^\circ$       **c.**  $\tan 500^\circ$

## ? Example 5 – Angles in all four quadrants

Find the exact value of  $\sin (-210^\circ)$ .

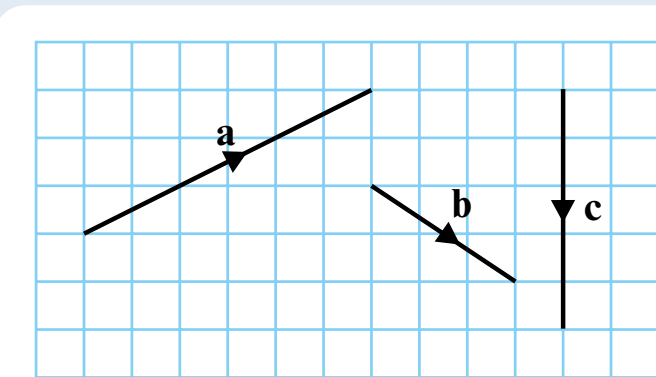


# Chapter 11 – Vectors

## Example 1 – Adding vectors

The diagram shows vectors  $\mathbf{a}$ ,  $\mathbf{b}$  and  $\mathbf{c}$ .

Draw a diagram to illustrate the vector addition  $\mathbf{a} + \mathbf{b} + \mathbf{c}$ .

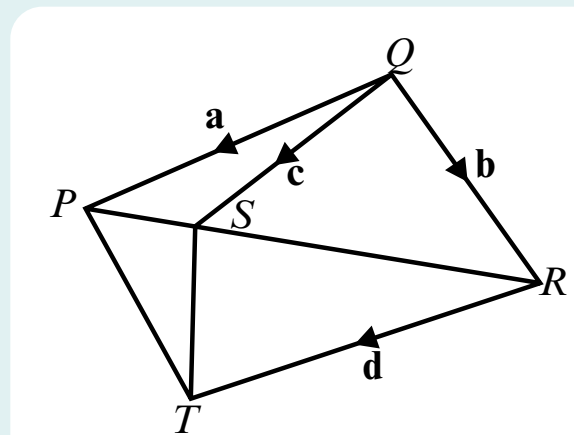


## Example 2 – Adding vectors

In the diagram,  $\vec{QP} = \mathbf{a}$ ,  $\vec{QR} = \mathbf{b}$ ,  $\vec{QS} = \mathbf{c}$  and  $\vec{RT} = \mathbf{d}$

Find in terms of  $\mathbf{a}$ ,  $\mathbf{b}$ ,  $\mathbf{c}$  and  $\mathbf{d}$ :

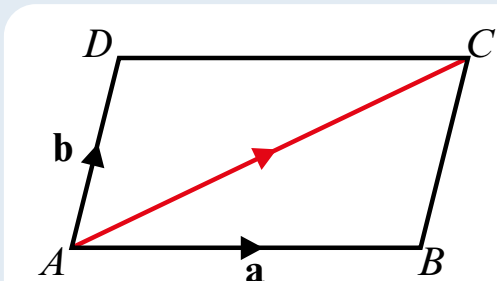
- $\vec{PS}$
- $\vec{RP}$
- $\vec{PT}$
- $\vec{TS}$



## Example 3 – Parallelograms and vectors

$ABCD$  is a parallelogram.

$\vec{AB} = \mathbf{a}$ ,  $\vec{AD} = \mathbf{b}$ . Find  $\vec{AC}$ .



## Example 4 – proving vectors are parallel

Show that the vectors  $6\mathbf{a} + 8\mathbf{b}$  and  $9\mathbf{a} + 12\mathbf{b}$  are parallel.

## Example 5 – Midpoints and vectors

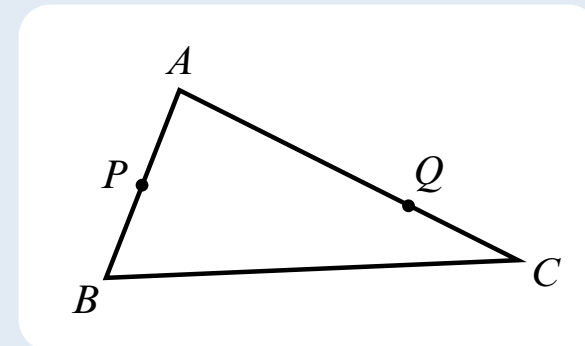
In triangle  $ABC$ ,  $\vec{AB} = \mathbf{a}$  and  $\vec{AC} = \mathbf{b}$ .

$P$  is the midpoint of  $AB$ .

$Q$  divides  $AC$  in the ratio 3 : 2.

Write in terms of  $\mathbf{a}$  and  $\mathbf{b}$ :

- $\vec{BC}$
- $\vec{AP}$
- $\vec{AQ}$
- $\vec{PQ}$



## Example 6 – Adding and subtracting vectors in column form

$$\mathbf{a} = \begin{pmatrix} 2 \\ 6 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$$

Find **a.**  $\frac{1}{3}\mathbf{a}$       **b.**  $\mathbf{a} + \mathbf{b}$       **c.**  $2\mathbf{a} - 3\mathbf{b}$

## Example 7 – Adding and subtracting vectors in component form

$$\mathbf{a} = 3\mathbf{i} - 4\mathbf{j}, \mathbf{b} = 2\mathbf{i} + 7\mathbf{j}$$

Find **a.**  $\frac{1}{2}\mathbf{a}$       **b.**  $\mathbf{a} + \mathbf{b}$       **c.**  $3\mathbf{a} - 2\mathbf{b}$

## Example 8 – Representing vectors

- Draw a diagram to represent the vector  $-3\mathbf{i} + \mathbf{j}$
- Write this as a column vector.

## Example 9 – Adding vectors in column form

Given that  $\mathbf{a} = 2\mathbf{i} + 5\mathbf{j}$ ,  $\mathbf{b} = 12\mathbf{i} - 10\mathbf{j}$  and  $\mathbf{c} = -3\mathbf{i} + 9\mathbf{j}$ , find  $\mathbf{a} + \mathbf{b} + \mathbf{c}$ , using column vector notation in your working.

## Example 10 – Adding vectors in component form

Given  $\mathbf{a} = 5\mathbf{i} + 2\mathbf{j}$  and  $\mathbf{b} = 3\mathbf{i} - 4\mathbf{j}$ , find  $2\mathbf{a} - \mathbf{b}$  in terms of  $\mathbf{i}$  and  $\mathbf{j}$ .

## Example 11 – Vector magnitude

Given that  $\mathbf{a} = 3\mathbf{i} + 4\mathbf{j}$  and  $\mathbf{b} = -2\mathbf{i} - 4\mathbf{j}$ :

- find  $|\mathbf{a}|$
- find a unit vector in the direction of  $\mathbf{a}$
- find the exact value of  $|2\mathbf{a} + \mathbf{b}|$

## Example 12 – Vector direction

Find the angle between the vector  $4\mathbf{i} + 5\mathbf{j}$  and the positive  $x$ -axis.

## Example 13 – Vector direction

Vector  $\mathbf{a}$  has magnitude 10 and makes an angle of  $30^\circ$  with  $\mathbf{j}$ .

Find  $\mathbf{a}$  in  $\mathbf{i}$ ,  $\mathbf{j}$  and column vector format.

