Year 11 to Year 12 Transition Paper

Trigonometric Ratios

Mark Scheme

Question	Scheme	Marks
1 (Way 1)	$\cos 63 = \frac{24.3}{(PQ)}$ **or** $\sin 27 = \frac{24.3}{(PQ)}$ **or** $\frac{(PQ)}{\sin 90} = \frac{24.3}{\sin 27}$ **or** $\frac{\sin 90}{(PQ)} = \frac{\sin 27}{24.3}$ oe	M1
	$(PQ =)\ \frac{24.3}{\cos 63}$ **or** $(PQ =)\ \frac{24.3}{\sin 27}$ **or** $(PQ) = \frac{24.3}{\sin 27} \times \sin 90$	M1
	53.5	A1
(Way 2)	$(RQ =) 24.3 \times \tan 63 (= 47.6914..)$	M1
	$(PQ =) \sqrt{'47.6914'^2 + 24.3^2}$ oe	M1
	53.5	A1
		(3 marks)

Notes
Way 1
M1 for a correct trigonometric ratio
M1 for a correct rearrangement for *PQ*

Both
A1 Accept 53.5 - 53.53

Question	Scheme		Marks
2(a)	Finds third angle of triangle and uses or states $\frac{x}{\sin 60°} = \frac{30}{\sin"50°"}$	Finds third angle of triangle and uses or states Finds third angle of triangle and uses or states $\frac{y}{\sin 70°} = \frac{30}{\sin"50°"}$	M1
	So $x = \frac{30\sin 60°}{\sin 50°}$ $(= 33.9)$	So $y = \frac{30\sin 70°}{\sin 50°}(= 36.8)$	A1
	Area = $\frac{1}{2} \times 30 \times x \times \sin 70°$ or $\frac{1}{2} \times 30 \times y \times \sin 60$		M1
	$= 478 \text{ m}^2$		A1 ft
			(4)
(b)	Plausible reason, e.g. Because the angles and the side length are not given to four significant figures Or e.g. The lawn may not be flat		B1
			(1)
			(5 marks)

Notes

(a) M1: Uses sine rule with third angle to find one of the unknown side lengths
A1: finds expression for, or value of either side length
M1: Completes method to find area of triangle
A1 ft: Obtains a correct answer for their value of x or their value of y.

(b) B1: As information given in the equation may not be accurate to 4sf or the lawn may not be flat so modelling by a plane figure may not be accurate.

Question	Scheme	Marks
3(a)	Uses $18\sqrt{3}=\frac{1}{2}\times 2x\times 3x\times\sin 60°$	M1
	Sight of $\sin 60°=\frac{\sqrt{3}}{2}$ and proceeds to $x^2=k$ oe	M1
	$x=\sqrt{12}=2\sqrt{3}$ *	A1*
		(3)
(b)	Uses $BC^2=\left(6\sqrt{3}\right)^2+\left(4\sqrt{3}\right)^2-2\times 6\sqrt{3}\times 4\sqrt{3}\times\cos 60°$	M1
	$BC^2=84$	A1
	$BC=2\sqrt{21}$ (cm)	A1
		(3)
		(6 marks)

Notes

(a)

M1: Attempts to use the formula $A=\frac{1}{2}ab\sin C$.

If the candidate writes $18\sqrt{3}=\frac{1}{2}\times 5x\times\sin 60°$ **without** sight of a previous correct line then this would be M0

M1: Sight of $\sin 60°=\frac{\sqrt{3}}{2}$ or awrt 0.866 and proceeds to $x^2=k$ oe such as $px^2=q$

This may be awarded from the correct formula or $A=ab\sin C$

A1*: Look for $x^2=12\Rightarrow x=2\sqrt{3}$, $x^2=4\times 3\Rightarrow x=2\sqrt{3}$ or $x=\sqrt{12}=2\sqrt{3}$

This is a given answer and all aspects must be correct including one of the above intermediate lines. It cannot be scored by using decimal equivalents to $\sqrt{3}$

Alternative using the given answer of $x=2\sqrt{3}$

M1: Attempts to use the formula $A=\frac{1}{2}\times 4\sqrt{3}\times 6\sqrt{3}\sin 60°$ oe

M1: Sight of $\sin 60°=\frac{\sqrt{3}}{2}$ and proceeds to $A=18\sqrt{3}$

A1*: Concludes that $x=2\sqrt{3}$

(b)

M1: Attempts the cosine rule with the sides in the correct position.

This can be scored from $BC^2=(3x)^2+(2x)^2-2\times 3x\times 2x\times\cos 60°$ as long as there is some attempt to substitute x in later. Condone slips on the squaring

A1: $BC^2=84$ Accept $BC^2=7\times 12$, $BC=\sqrt{84}$ or $BC=2\sqrt{21}$

If they replace the surds with decimals they can score the A1 for $BC^2=$ awrt 84.0

A1: $BC=2\sqrt{21}$

Condone other variables, say $x=2\sqrt{21}$, but it cannot be scored via decimals.

Question	Scheme	Marks
4	250 = 0.5 × 26 × *AC* × sin(39) oe	M1
	(*AC*=) 30.5(5579...) or 30.6	A1
	$\frac{(AB)}{\sin 47} = \frac{'30.56'}{\sin 95}$ oe **or** $\frac{(BC)}{\sin(180-95-47)} = \frac{'30.56'}{\sin 95}$ oe	M1
	$(AB =)\frac{'30.56'}{\sin 95} \times \sin 47$ (= 22.4(3407...)) **or** $(BC =)\frac{'30.56'}{\sin 95} \times \sin(180-95-47)$ (= 18.8(8524...))	M1
	250 + 0.5 × '30.56' × '22.43' × sin(180 – 95 – 47) (= 461.03....) **or** 250 + 0.5 × '30.56' × '18.88' × sin(47) (= 461.03....)	M1
	461	A1
		(6 marks)

Notes

M1 for using the area formula correctly. If this mark is awarded then ft on the remaining M marks
M1 dep on M1 for correct substitution into sine rule
M1 (dep on previous M marks) for a correct method to find a missing length or sight of values in the ranges
22.39 – 22.47 for AB
18.8 – 18.92 for BC
M1 for a complete method to find total area
A1 accept 461 - 462

Question	Scheme	Marks
5	($AC^2 =$) $4.1^2 + 5.3^2 - 2 \times 4.1 \times 5.3 \times \cos(110)$	M1
	($AC =$) $\sqrt{16.81 + 28.09 + 14.8(641...)}$ or $\sqrt{59.7(641...)}$ or 7.7(3073) **or** $AC^2 = 59.7\ldots$	M1
	Eg $\frac{\sin x}{5.3} = \frac{\sin 110}{"7.7"}$ **or** $\frac{5.3}{\sin x} = \frac{"7.7"}{\sin 110}$ **or** $5.3^2 = 4.1^2 + "7.7"^2 - 2\times4.1\times"7.7"\times\cos x$ oe	M1
	Eg $\sin x = \frac{\sin 110}{"7.7"} \times 5.3$ (= 0.644(2…)) **or** $\cos x = \frac{4.1^2 + "7.7"^2 - 5.3^2}{2\times 4.1 \times "7.7"}$ (= 0.764(83…) (= 22.4(3407...)) **or** $(BC =)\frac{'30.56'}{\sin 95}\times\sin(180-95-47)$ (= 18.8(8524...))	M1
	40.1	A1
		(6 marks)

Notes

M1 for the correct use of Cosine rule to find AC
M1 NB: there must be evidence of correct order of operations for this mark to be awarded
M1dep on first M1
for correct use of sine rule or cosine rule ft for their value of AC or AC2
M1for isolating sinx or cosx
A1for 40.1 – 40.11

Question	Scheme	Marks
6(a)	Attempts $\dfrac{\sin \angle ACB}{6.5} = \dfrac{\sin 35}{4.7}$	M1
	$\angle ACB = \text{awrt}(52 \text{ or } 53)°$ or awrt$(127 \text{or} 128)°$	A1
	$\angle ACB = 127.5°$	A1
		(3)
(b)	Eg $\dfrac{(AC)}{\sin"17.5°"} = \dfrac{6.5}{\sin"127.5°"}$ or $= \dfrac{4.7}{\sin 35°}$	M1
	$\left[\dfrac{(CD)}{\sin"75°"} = \dfrac{4.7}{\sin"127.5°"} \Rightarrow (CD) = ... \Rightarrow (AC)+(CD)\right]$=awrt 8.2	A1
	Total length of wood $= 8.1 + 6.5 + 4.7 + 4.7 = \text{awrt} 24.1$	A1
		(3)
		(6 marks)
Alt1(a)	$\cos 35 = \dfrac{AC^2 + 6.5^2 - 4.7^2}{2 \times 6.5 \times AC} \Rightarrow AC^2 - 13\cos(35)AC + 20.16 = 0 \Rightarrow AC = ...$ $\cos \angle ACB = \dfrac{"AC"^2 + 4.7^2 - 6.5^2}{2 \times "AC" \times 4.7}$ oe	M1

Question	Scheme	Marks
7 (a)	Area *ABCD* is 40 cm^2 $\Rightarrow 40 = 6\times 10\sin\theta$ oe	M1
	$\sin\theta = \frac{2}{3} \Rightarrow \theta = 180° - 41.8°$	M1
	$\angle DAB = \text{awrt } 138.19°$	A1
		(3)
(b)	Attempts $DB^2 = 10^2 + 6^2 - 2\times 10\times 6\cos\text{"}138.19°\text{"}$	M1
	$DB = \text{awrt } 15.0$ (cm)	A1
		(2)
		(5 marks)

Notes

(a)

M1 Scored for a correct attempt at using the area of *ABCD* is 40 cm^2

Score for $40 = 6\times 10\times\sin\theta$ or $20 = \frac{1}{2}\times 6\times 10\times\sin\theta$ where θ is one of the corner angles.

M1 Score for $\sin\theta = k \Rightarrow \theta = 180° - \arcsin k$

A1 $\angle DAB = \text{awrt } 138.19°$

(b)

M1 Attempts $DB^2 = 10^2 + 6^2 - 2\times 10\times 6\cos\text{"}138.19°\text{"}$ - allow if the angle used is acute as long as it is clearly their attempt at angle *DAB*. So allow use of $41.8°$ unless they have correctly found angle *DAB* and chosen the wrong one here.

A1 $DB = \text{awrt } 15.0$ (cm) Accept 15 in place of 15.0. Allow from attempts using awrt 138°

………
…

Alt for (a)

M1 Area *ABCD* is 40 cm^2 $\Rightarrow h = \frac{40}{10} = ... \Rightarrow \sin\angle ABC = \frac{\text{"}4\text{"}}{6}$ OR $\cos\angle ABC = \frac{\text{"}4\text{"}}{6}$ oe

Essentially this mark is for using the area together with an appropriate trig identity to form an equation in the sine or cosine of one of the angles of the parallelogram. Attempts finding "*DX*" where *X* is where the perpendicular to *DC* through *A* meets *DC* are possible.

M1 $\angle DAB = 180° - \arcsin\left(\frac{\text{"}4\text{"}}{6}\right) = ...$ or may see $\angle DAB = 90° + \arccos\left(\frac{\text{"}4\text{"}}{6}\right) = ...$

This is for a complete correct method to find the angle *DAB*

A1 $\theta = \text{awrt } 138.19°$

Question	Scheme	Marks
8(a)	$\left(0, -\frac{\sqrt{3}}{2}\right)$	B1
	and (60°, 0) and (240°, 0) and (-120°, 0) and (-300°, 0)	B1 B1
		(3)
(b)	$\sin(x-60°)=\frac{\sqrt{6}-\sqrt{2}}{4}$ (= .2588)	M1
	x – 60° = 15° (or 165° or – 195° or – 345°) or 0.262 or $\frac{\pi}{12}$ radians	A1
	So x = 75° or 225° or – 135° or –285° (allow awrt)	M1 A1 A1
		(5)
		8 marks

Notes

(a) **B1** : Correct exact y intercept (not decimal) – allow on the diagram or in the text. Allow $y=-\frac{\sqrt{3}}{2}$

B1 for 2 correct x intercepts then **third B1** for all 4 correct x intercepts (may or may not be given as coordinates – may be given on graph) Must be in degrees. (Extra answers in the range lose the **third B1**)

(b) **M1**: Divides by 4 first giving correct statement $\sin(x-60°)=\frac{\sqrt{6}-\sqrt{2}}{4}$ but

$(x-60°)=\frac{\sqrt{6}-\sqrt{2}}{4}$ **is M0 and** $\sin x-\sin 60°=\frac{\sqrt{6}-\sqrt{2}}{4}$ **is also M0 and**

$\sin(x-60°)=\frac{\sqrt{4}}{4}$ **is M0** if not preceded by correct statement

A1: Obtains 15° (or 165° or – 195° or – 345°)
M1: Adds 60° to their previous answer which should have been in degrees and obtained by using inverse sine
A1: Two correct answers second **A1:** All four correct answers Extra answers outside range are ignored. Lose final A mark for extra wrong answers in the range.
If they approximate too early allow awrt answers given for full marks. (e.g. 75.01 etc)
Answers in mixture, degrees and radians: Allow first M A1 only so M1A1M0A0A0 for 60.262 for example

Question	Scheme		Marks
9(a)	$\frac{\sin ACB}{4x}=\frac{\sin 30°}{3x}$	Attempts the sine rule with the sides and angles in the correct places	M1
	$\sin ACB=\frac{0.5\times 4x}{3x}=\frac{2}{3}*$	Proceeds without errors to given answer with at least one intermediate line of working.	A1*
			(2)
(a) Way 2	$\frac{\frac{2}{3}}{4x}=\frac{\sin 30°}{3x}\Rightarrow\frac{\frac{2}{3}}{4x}=\frac{\frac{1}{2}}{3x}$	Attempts the sine rule with the sides and angles in the correct places and replaces sin *ACB* by 2/3 and sin 30 by 1/2	M1
	$2x=2x$ so $\sin ACB=\frac{2}{3}$	Correct working to achieve both sides equal **and** conclusion	A1
	Notes: Score M1A1 for $\sin ACB=\frac{4\sin 30°}{3}=\frac{2}{3}$ Score M1A0 for $\frac{\sin ACB}{4x}=\frac{\sin 30°}{3x}\Rightarrow ACB=41.81...\Rightarrow\sin ACB=\frac{2}{3}$ Score M0A0 for $ACB=41.81...\Rightarrow\sin ACB=\frac{2}{3}$ (no sin rule used)		
			(2)
(b)	(Obtuse $ACB=$) $180-\left(\sin^{-1}\left(\frac{2}{3}\right)\right)$ Attempts to find obtuse *ACB* but ignore how it is referenced i.e. just look for an attempt at the calculation		M1
	(Angle $ABC=$) awrt $11.81°$	Awrt $11.81°$ **(Must be seen in (b))**	A1
			(2)
	Note that in (c) and (d), the M marks are available for using *ABC* as 41.81… if the candidate clearly thinks that this is *ABC* – this may be seen labelled on the diagram at *B* or is clearly their answer to part (b)		
(c)	$20=\frac{1}{2}4x\times 3x\times\sin'11.81'$	Attempts to use Area of triangle formula $\frac{1}{2}ab\sin C$ with A = 20, $4x$, $3x$ and their $11.81°$	M1
	$x^2=16.29$	Proceeds using correct arithmetic and fully correct processing to $x^2=$ … **Dependent on previous mark.**	**d**M1
	$x=4.04$	Awrt 4.04	A1

Question	Scheme	Marks
		(3)

(d)	Attempts the cosine rule **to obtain a value for *AC***: $AC^2 = (4\times"4.04")^2 + (3\times"4.04")^2 - 2\times(4\times"4.04")(3\times"4.04")\cos("11.81")$ $\Rightarrow AC = ...$ Condone poor bracketing e.g. $4\times"4.04"^2$ rather than $(4\times"4.04")^2$ Or uses area **to obtain a value for *AC***: Uses $\frac{1}{2}\times 4"x"\times AC\sin 30° = 20 \Rightarrow AC = ...$ Or sine rule **to obtain a value for *AC***: $\frac{AC}{\sin"11.81"} = \frac{3\times"x"}{\sin 30°} \Rightarrow AC = ...$ or $\frac{AC}{\sin"11.81"} = \frac{4\times"x"}{\sin(TheirACB)} \Rightarrow AC = ...$	M1
	$\Rightarrow AC = 4.96$ Awrt 4.96 (**allow also awrt 4.95**) This comes from $\frac{1}{2}\times 4"x"\times AC\sin 30° = 20 \Rightarrow AC = \frac{20}{x} = \frac{20}{4.04} = 4.95...$	A1
		(2)
		(9 marks)

Question	Scheme	Marks
10 (a)	Uses $15 = \frac{1}{2} \times 5 \times 10 \times \sin\theta$	M1
	$\sin\theta = \frac{3}{5}$ oe	A1
	Uses $\cos^2\theta = 1 - \sin^2\theta$	M1
	$\cos\theta = \pm\frac{4}{5}$	A1
		(4)
(b)	Uses $BC^2 = 10^2 + 5^2 - 2 \times 10 \times 5 \times "-\frac{4}{5}"$	M1
	$BC = \sqrt{205}$	A1
		(2)
		(6 marks)

Notes

(a)

M1: Uses the formula $\text{Area} = \frac{1}{2}ab\sin C$ in an attempt to find the value of $\sin\theta$ or θ

A1: $\sin\theta = \frac{3}{5}$ oe This may be implied by $\theta = \text{awrt } 36.9°$ or awrt 0.644 (radians)

M1: Uses their value of $\sin\theta$ to find two values of $\cos\theta$ This may be scored via the formula $\cos^2\theta = 1 - \sin^2\theta$ or by a triangle method. Also allow the use of a graphical calculator or candidates may just write down the **two values**. The values must be symmetrical $\pm k$

A1: $\cos\theta = \pm\frac{4}{5}$ or ± 0.8 Condone these values appearing from $\pm 0.79....$

(b)

M1: Uses a suitable method of finding the longest side. For example chooses the negative value (or the obtuse angle) and proceeds to find *BC* using the cosine rule. Alternatively works out *BC* using both values and chooses the larger value. If stated the cosine rule should be correct (with a minus sign). Note if the sign is +ve and the acute angle is chosen the correct value will be seen. It is however M0 A0

A1: $BC = \sqrt{205}$