Year 11 to Year 12 Transition Paper

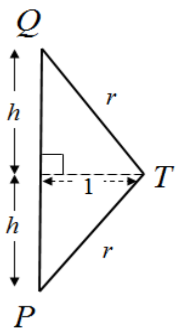
Circles

Mark Scheme

Question	Working	Answer	Mark	Notes
1		Circle radius 4 Centre (3,0) and (−1,0) and (7,0) labelled	M1	For centre (3,0) implied by drawing or label or a circle of radius 4 or intersections on the *x*-axis at −1or 7 implied by drawing or labels
			M1	for 2 of centre (3,0) implied by drawing or label intersections on the *x*-axis at −1 and 7 implied by drawing or label circle drawn with radius 4
			A1	for a fully correct answer
				(3 marks)

Question	Scheme	Marks
2(a)	$\{PQ=\}\ \sqrt{(7-10)^2+(8-13)^2}$ or $\sqrt{(10-7)^2+(13-8)^2}$	M1
	$\{PQ\}=\sqrt{34}$	A1
		(2)
(b)	$(x-7)^2+(y-8)^2=34$ $\left(\text{or } \left(\sqrt{34}\right)^2\right)$	M1
		A1 oe
		(2)
(c)	$\{\text{Gradient of radius}\}=\dfrac{13-8}{10-7}$ or $\dfrac{5}{3}$	B1
	Gradient of tangent $=-\dfrac{1}{m}\left(=-\dfrac{3}{5}\right)$	M1
	$y-13=-\dfrac{3}{5}(x-10)$	M1
	$3x+5y-95=0$	A1
		(4)
		(8 marks)

Question	Scheme	Marks
3	**You may mark (a) and (b) together** $x^2 + y^2 - 2x + 14y = 0$	
(a)	Obtain LHS as $\underline{(x \pm 1)^2} + \underline{\underline{(y \pm 7)^2}} = \ldots$	M1
	Centre is $(1, -7)$.	A1 **(2)**
(b)	Uses $r^2 = a^2 + b^2$ or $r = \sqrt{a^2 + b^2}$ where their centre was at $(\pm a, \pm b)$ $r = \sqrt{50}$ or $5\sqrt{2}$	M1 A1 **(2)**
(c)	Substitute $x = 0$ in either form of equation of circle and solve resulting quadratic to give $y =$ $y^2 + 14y = 0$ *so* $y = 0$ *and* -14 **or** $\underline{\underline{(y \pm 7)^2 - 49}} = 0$ *so* $y = 0$ *and* -14	M1 A1 **(2)**
(d)	Gradient of radius joining centre to (2,0) is $\frac{"-7"-0}{"1"-2}$ $(= 7)$ Gradient of tangent is $\frac{-1}{m}$ $(= -\frac{1}{7})$ So equation is $y - 0 = -\frac{1}{7}(x - 2)$ and so $x + 7y - 2 = 0$	M1 M1 M1, A1 **(4)**
		(10 marks)
Alternative Methods which may be seen		
(a)	Method 2: Comparing with $x^2 + y^2 + 2gx + 2fy + c = 0$ to write down centre $(-g, -f)$ directly. Condone sign errors for this M mark**.** Centre is $(1, -7)$.	**M1** **A1** **(2)**
(b)	Method 2: Using $\sqrt{g^2 + f^2 - c}$. So $r = \sqrt{50}$ or $5\sqrt{2}$	**M1 A1** **(2)**
(d)	Method 3: Using Implicit Differentiation $2x + 2y\frac{dy}{dx} - 2 + 14\frac{dy}{dx} = 0$ **or** $2(x-1) + 2(y+7)\frac{dy}{dx} = 0$	**M1**
	$\frac{dy}{dx} = \ldots\left(\frac{2-2x}{14+2y} = \frac{-2}{14}\right)$	**M1**
	So equation is $y - 0 = -\frac{1}{7}(x - 2)$ and so $x + 7y - 2 = 0$	**M1, A1** **(4)**
	Method 4: Making *y* the subject of the formula and differentiating	
	$y = -7 \pm \sqrt{\{50 - (x-1)^2\}}$ so $\frac{dy}{dx} = \pm\frac{1}{2} \times -2(x-1)\{50 - (x-1)^2\}^{-\frac{1}{2}}$	**M1**
	At $x = 2$, $\frac{dy}{dx} = \mp\frac{1}{7}$	**M1**
	So equation is $y - 0 = \mp\frac{1}{7}(x - 2)$	**M1**
	Chooses $\frac{dy}{dx} = -\frac{1}{7}$ and so $x + 7y - 2 = 0$	**A1**

Question	Scheme	Marks
4(a)	$$x^2+y^2-10x+6y+30=0$$	
	Uses any appropriate method to find the coordinates of the centre, e.g achieves $\underline{(x\pm5)^2}+\underline{\underline{(y\pm3)^2}}=\ldots$. Accept $(\pm5,\pm3)$ as indication of this.	M1
	Centre is $(5,-3)$.	A1 **(2)**
(b) **Way 1**	Uses $\underline{(x\pm"5")^2-"5^2"}+\underline{\underline{(y\pm"3")^2-"3^2"}}+30=0$ to give $r=\sqrt{"25"+"9"-30}$ or $r^2="25"+"9"-30$ (not $30-25-9$)	M1
	$r=2$	A1cao **(2)**
Or Way 2	Using $\sqrt{g^2+f^2-c}$ from $x^2+y^2+2gx+2fy+c=0$ (Needs formula stated or correct working)	M1
	$r=2$	A1 **(2)**
(c) **Way 1**	Use $x=4$ in *an* equation of circle and obtain equation in y only	M1
	e.g $(4-5)^2+(y+3)^2=4$ or $4^2+y^2-10\times4+6y+30=0$	
	Solve their quadratic in y and obtain **two** solutions for y	dM1
	e.g. $(y+3)^2=3$ or $y^2+6y+6=0$ so $y=-3\pm\sqrt{3}$	A1 **(3)**
Or Way 2	 Divide triangle PTQ and use Pythagoras with $"r"^2-("5"-4)^2=h^2$,	M1
	Find h and evaluate $"-3"\pm h$. May recognise $(1,\sqrt{3},2)$ triangle.	dM1
	So $y=-3\pm\sqrt{3}$	A1 **(3)**
		(7 marks)

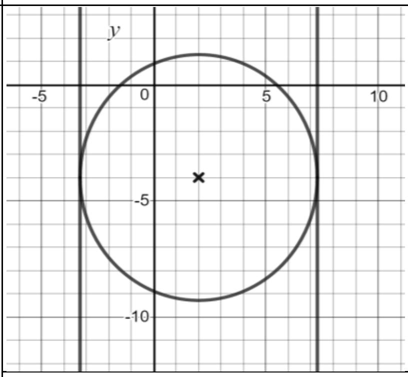
Question	Scheme	Marks
5	The equation of the circle is $(x+1)^2+(y-7)^2=(r^2)$	M1 A1
	The radius of the circle is $\sqrt{(-1)^2+7^2}=\sqrt{50}$ or $5\sqrt{2}$ or $r^2=50$	M1
	So $(x+1)^2+(y-7)^2=50$ or equivalent	A1
		(4 marks)

Question	Scheme	Marks
6 (a)	$(x\mp 2)^2+(y\pm 1)^2=k$, $k>0$	M1
	Attempts to use $r^2=(4-2)^2+(-5+1)^2$	M1
	Obtains $(x-2)^2+(y+1)^2=20$	A1
		(3)
(b)	Gradient of radius from centre to (4, –5) = –2 (must be correct)	B1
	Tangent gradient = $-\dfrac{1}{\text{their numerical gradient of radius}}$	M1
	Equation of tangent is $(y+5)='\tfrac{1}{2}'(x-4)$	M1
	So equation is $x-2y-14=0$	A1
		(4)
		(7 marks)

Question	Scheme		Marks
7(a) (i)	The centre is at (10, 12)	B1: $x = 10$	B1 B1
(ii)		B1: $y = 12$	
	Uses $(x-10)^2 + (y-12)^2 = -195+100+144 \Rightarrow r = ...$		M1
	Completes the square for both *x* and *y* in an attempt to find *r*. $(x \pm "10")^2 \pm a$ and $(y \pm "12")^2 \pm b$ and $+195 = 0, (a, b \neq 0)$ Allow errors in obtaining their r^2 but must find square root		
	$r = \sqrt{10^2 + 12^2 - 195}$	A correct numerical expression for *r* including the square root and can implied by a correct value for *r*	A1
	$r = 7$	Not $r = \pm 7$ unless -7 is rejected	A1
			(5)
(b)	$MN = \sqrt{(25-"10")^2 + (32-"12")^2}$	Correct use of Pythagoras	M1
	$MN \left(= \sqrt{625}\right) = 25$		A1
			(2)
(c)	$NP = \sqrt{("25"^2 - "7"^2)}$	$NP = \sqrt{(MN^2 - r^2)}$	M1
	$NP \left(= \sqrt{576}\right) = 24$		A1
			(2)
			(9 marks)

Question	Scheme		Marks
8(a)	$C\left(\frac{-2+8}{2}, \frac{11+1}{2}\right) = C(3, 6)$ **AG**	Correct method (no errors) for finding the mid-point of AB giving $(3, 6)$	B1*
			(1)
(b)	$(8-3)^2 + (1-6)^2$ or $\sqrt{(8-3)^2 + (1-6)^2}$ or $(-2-3)^2 + (11-6)^2$ or $\sqrt{(-2-3)^2 + (11-6)^2}$	Applies distance formula in order to find the radius.	M1
		Correct application of formula.	A1
	$(x-3)^2 + (y-6)^2 = 50 \left(\text{or } \left(\sqrt{50}\right)^2 \text{ or } \left(5\sqrt{2}\right)^2\right)$	$(x \pm 3)^2 + (y \pm 6)^2 = k$, k is a positive <u>value</u>.	M1
		$(x-3)^2 + (y-6)^2 = 50$ (Not 7.07^2)	A1
			(4)
(c)	{For $(10, 7)$, } <u>$(10-3)^2 + (7-6)^2 = 50$</u>, {so the point lies on C.}		<u>B1</u>
			(1)
(d)	{Gradient of radius} $= \frac{7-6}{10-3}$ or $\frac{1}{7}$	This must be seen in part (d).	B1
	Gradient of tangent $= \frac{-7}{1}$	Using a perpendicular gradient method.	M1
	$y - 7 = -7(x - 10)$	$y - 7 = (\text{their gradient})(x - 10)$	M1
	$y = -7x + 77$	$y = -7x + 77$ or $y = 77 - 7x$	A1 cao
			(4)
			(10 marks)

Question	Scheme	Marks
9(a)	Obtain $\underline{(x \pm 10)^2}$ **and** $\underline{\underline{(y \pm 8)^2}}$	M1
	Obtain $\underline{(x - 10)^2}$ **and** $\underline{\underline{(y - 8)^2}}$	A1
	Centre is (10, 8). N.B. This may be indicated on **diagram only** as (10, 8)	A1 **(3)**
(b)	See $\underline{(x \pm 10)^2} + \underline{\underline{(y \pm 8)^2}} = 25 (= r^2)$ or $(r^2 =)"100"+"64"-139$	M1
	$r = 5$ * (this is a printed answer so need one of the above two reasons)	A1 **(2)**
(c)	Use x = 13 in either form of equation of circle and solve resulting quadratic to give y =	M1
	e.g $x = 13 \Rightarrow (13-10)^2 + (y-8)^2 = 25 \Rightarrow (y-8)^2 = 16$ so y=	
	$or\ 13^2 + y^2 - 20\times 13 - 16y + 139 = 0 \Rightarrow y^2 - 16y + 48 = 0$ so y =	
	y = 4 or 12 (on EPEN mark one correct value as A1A0 and both correct as A1 A1)	A1, A1 **(3)**
		(8 marks)

Question	Scheme	Marks
10(a)	$x^2+y^2-4x+8y-8=0$	
	Attempts $(x-2)^2+(y+4)^2-4-16-8=0$	M1
	(i) Centre $(2,-4)$	A1
	(ii) Radius $\sqrt{28}$ oe Eg $2\sqrt{7}$	A1
		(3)
(b)	 Attempts to add/subtract '*r*' from '2'	M1
	$k=2\pm\sqrt{28}$	A1ft
		(2)
		(5 marks)

Question	Scheme		Marks
11(a)	$x^2 + y^2 + 4x - 2y - 11 = 0$		
	$\left\{ \underline{(x+2)^2 - 4} + \underline{\underline{(y-1)^2 - 1}} - 11 = 0 \right\}$	$(\pm 2, \pm 1)$, see notes.	M1
	Centre is $(-2, 1)$.	$(-2, 1)$.	A1 **cao** **(2)**
(b)	$(x+2)^2 + (y-1)^2 = 11 + 1 + 4$	$r = \sqrt{11 \pm "1" \pm "4"}$	M1
	So $r = \sqrt{11+1+4} \Rightarrow r = 4$	4 or $\sqrt{16}$ (Award A0 for ± 4).	A1 **(2)**
(c)	When $x = 0$, $y^2 - 2y - 11 = 0$	Putting $x = 0$ in C or their C.	M1
		$y^2 - 2y - 11 = 0$ or $(y-1)^2 = 12$, etc	A1 aef
	$y = \dfrac{2 \pm \sqrt{(-2)^2 - 4(1)(-11)}}{2(1)}$ $\left\{ = \dfrac{2 \pm \sqrt{48}}{2} \right\}$	Attempt to use formula or a method of completing the square in order to find $y = \ldots$	M1
	So, $y = 1 \pm 2\sqrt{3}$	$1 \pm 2\sqrt{3}$	A1 **cao** **cso** **(4)**
			(8 marks)

Question	Scheme	Marks
12(a)	$OQ^2 = \left(6\sqrt{5}\right)^2 + 4^2$ or $OQ = \sqrt{\left(6\sqrt{5}\right)^2 + 4^2}$ $\{= 14\}$	M1
	$y_Q = \sqrt{14^2 - 11^2}$	**d**M1
	$= \sqrt{75}$ or $5\sqrt{3}$	A1**cso**
		(3)
(b)	$(x - 11)^2 + \left(y - 5\sqrt{3}\right)^2 = 16$	M1A1
		(2)
		(5 marks)

Question	Scheme	Marks
13(a)	Equation of form $(x \pm 5)^2 + (y \pm 9)^2 = k$, $k > 0$	M1
	Equation of form $(x - a)^2 + (y - b)^2 = 5^2$, with values for *a* and *b*	M1
	$(x + 5)^2 + (y - 9)^2 = 25 = 5^2$	A1
		(3)
(b)	$P(8, -7)$. Let centre of circle $= X(-5, 9)$	
	$PX^2 = \left(8 - "-5"\right)^2 + \left(-7 - "9"\right)^2$ or $PX = \sqrt{\left(8 - -5\right)^2 + \left(-7 - 9\right)^2}$	M1
	($PX = \sqrt{425}$ or $5\sqrt{17}$) $PT^2 = \left(PX\right)^2 - 5^2$ with numerical *PX*	dM1
	$PT \left\{= \sqrt{400}\right\} = 20$ (allow 20.0)	A1 **cso**
		(3)
		(6 marks)

Question	Scheme	Marks
14(a)	Finds circle equation $(x \pm 2)^2 + (y \mp 6)^2 = (10 \pm (-2))^2 + (11 \mp 6)^2$	**M1**
	Checks whether (10, 1) satisfies their circle equation	**M1**
	Obtains $(x + 2)^2 + (y - 6)^2 = 13^2$ and checks that $(10 + 2)^2 + (1 - 6)^2 = 13^2$ and so states that (10, 1) lies on C (*)	**A1**
		(3)
(b)	Finds radius gradient $\frac{11-6}{10-(-2)}$ or $\frac{1-6}{10-(-2)}$ (m)	**M1**
	Finds gradient perpendicular to their radius using $-\frac{1}{m}$	**M1**
	Finds (equation and) y-intercept of tangent (see note below)	**M1**
	Obtains a correct value for y-intercept of their tangent, i.e. 35 or –23	**A1**
	Deduces gradient of second tangent	**M1**
	Finds (equation and) y-intercept of second tangent	**M1**
	So obtains distance PQ = 35 + 23 = 58 (*)	**A1**
		(7)
		(10 marks)

Question	Scheme	Marks
15(a)	Attempts to find the radius $\sqrt{(2--2)^2+(5-3)^2}$ or radius ^{2}	M1
	Attempts $(x-2)^2+(y-5)^2='r'^2$	M1
	Correct equation $(x-2)^2+(y-5)^2=20$	A1
		(3)
(b)	Gradient of radius OP where O is the centre of $C=\frac{5-3}{2--2}=\left(\frac{1}{2}\right)$	M1
	Equation of l is $-2=\frac{y-3}{x+2}$	dM1
	Any correct form $y=-2x-1$	A1
	Method of finding k Substitute $x=2$ into their $y=-2x-1$	M1
	$k=-5$	A1
		(5)
		(8 marks)

Question	Scheme	Marks
16(a)	Attempts to complete the square $(x \pm 3)^2 + (y \pm 5)^2 = ...$	M1
	(i) Centre $(3, -5)$	A1
	(ii) Radius 5	A1
		(3)
(b)	Uses a sketch or otherwise to deduce $k = 0$ is a critical value	B1
	Substitute $y = kx$ in $x^2 + y^2 - 6x + 10y + 9 = 0$	M1
	Collects terms to form correct 3TQ $(1+k^2)x^2 + (10k-6)x + 9 = 0$	A1
	Attempts $b^2 - 4ac ... 0$ for their *a*, *b* and *c* leading to values for *k* $"(10k-6)^2 - 36(1+k^2) ... 0" \rightarrow k = ..., ...$ $\left(0 \text{ and } \frac{15}{8}\right)$	M1
	Uses $b^2 - 4ac > 0$ and chooses the outside region (see note) for **their** critical values (Both *a* and *b* must have been expressions in *k*)	dM1
	Deduces $k < 0, k > \frac{15}{8}$ oe	A1
		(6)
		(9 marks)